How do planets help us understand stars?

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The wealth of Kepler data made it possible to study exoplanets-hosting stars. Many of the light curves that we get from Kepler or TESS are variable, often due to star spots, and other surface features that rotate into and out of view over the course of a stellar rotation. Using multiple light curves from Kepler, we are interested in mapping the surfaces of the exoplanet-hosting stars. Even though these stars are unresolved, we can in principle use their light curves to learn about what their surfaces look like by understanding the physical parameters of star spots. This can be efficiently done using hierarchical Bayesian modeling treating the star spots as a statistical population and using the exoplanet to put constraints on our statistical model. We are considering the properties of the star spots as hyperparameters of our Gaussian Process (GP) model. We use multiple stellar light curves to perform the ensemble statistics with the software Starry and its statistical adaptation, StarryProcess.

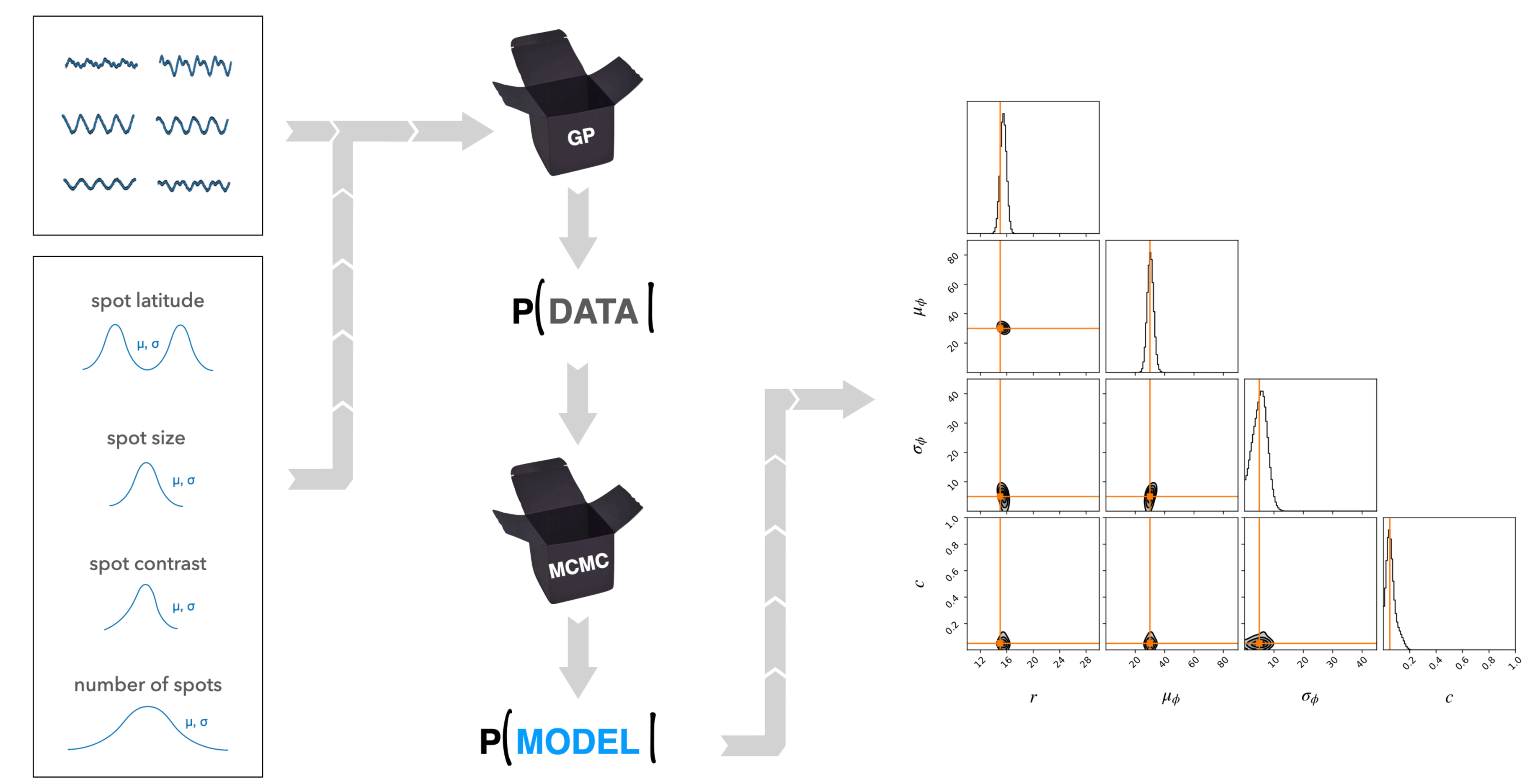
Starry is a Python package for modeling and analyzing the light curves and images of stars and exoplanets. The primary purpose of the package is to model the surfaces of stars and exoplanets as maps.

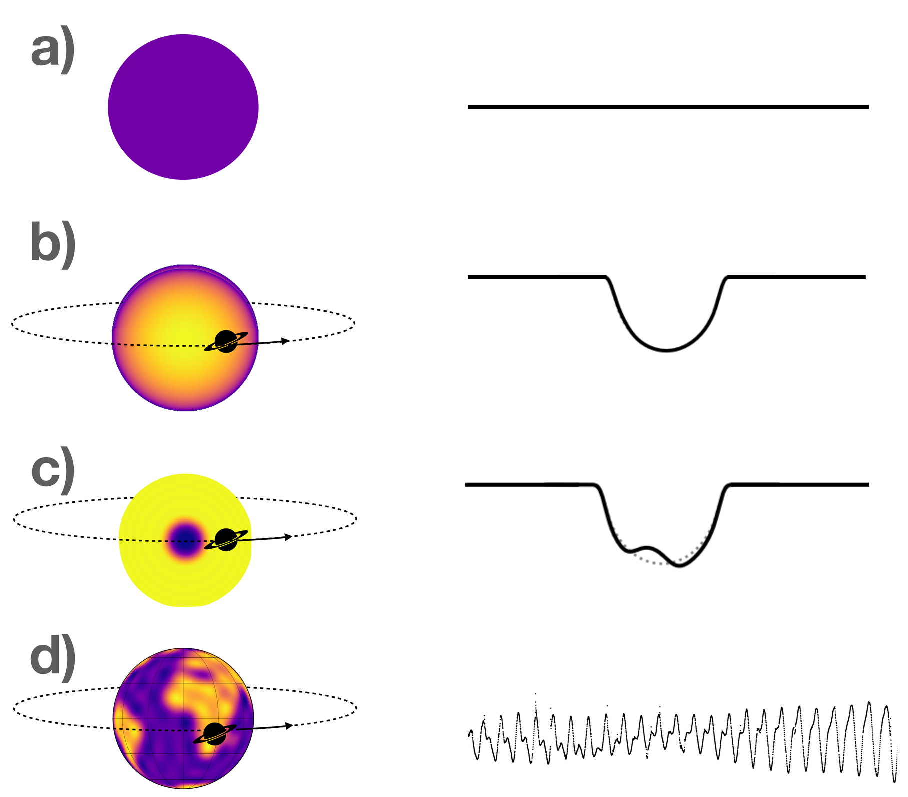
Methods:

$$P(\text{PLANET} \mid \text{DATA}) \sim \mathcal{N}(\text{DATA} - \text{MODEL}; \mu, \Sigma)\, P(\text{PLANET})$$

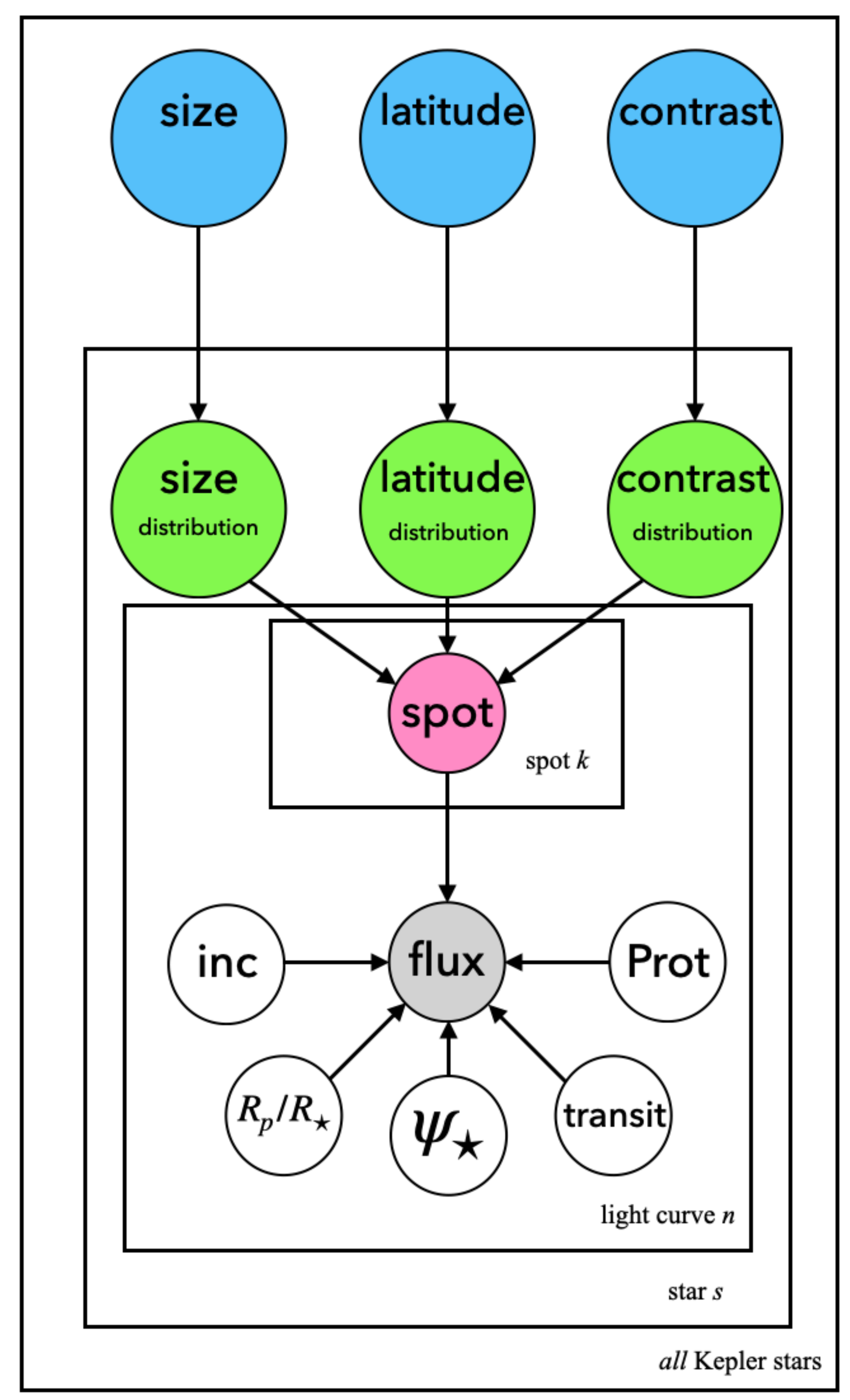
functions of things we might care about (or have prior knowledge about):

spot latitude distribution · spot size distribution · spot contrast distribution





An illustration of how light curves might look like in different cases: a) spotless star with no planet orbiting it; b) one planet orbiting a star; c) a planet orbiting a star with only one spot, and the planet crosses the spot (note, in this case, stellar rotation is assumed to be larger than planetary period); d) planet orbiting a star with a lot of spots results in a noisy light curve.



A schematic illustration of our graphical model.

Parameters:

We consider three sets of parameters: spot parameters, star's parameters, and planet's parameters:

$$\Theta = \left(\theta_\bullet, \theta_\star, \theta_p\right)$$

The spot parameters, which are also the hyperparameters of the GP kernel, include: number of spots, contrast, mean and standard deviation of the latitude distribution, and a radius of a spot:

$$\theta_\bullet = \left(n, c, \mu_\phi, \sigma_\phi, r\right)$$

The star parameters include stellar inclination, mass of a star, limb darkening coefficients, and rotational period:

$$\theta_\star = \left(i_\star, m_\star, u_1, u_2, P_\star\right)$$

The planet parameters are: inclination, eccentricity, spin-orbit misalignment angle, argument of pericenter, period, transit time, planet-to-star radii ratio:

$$\theta_p = \left(i_p, e, \psi, \omega, P, t_0, R_p/R_\star\right)$$

Given the GP prior and the likelihood function, we can calculate the joint posterior distribution over the hyperparameters Θ and the true function f_{true} given the observed data f_{obs} :

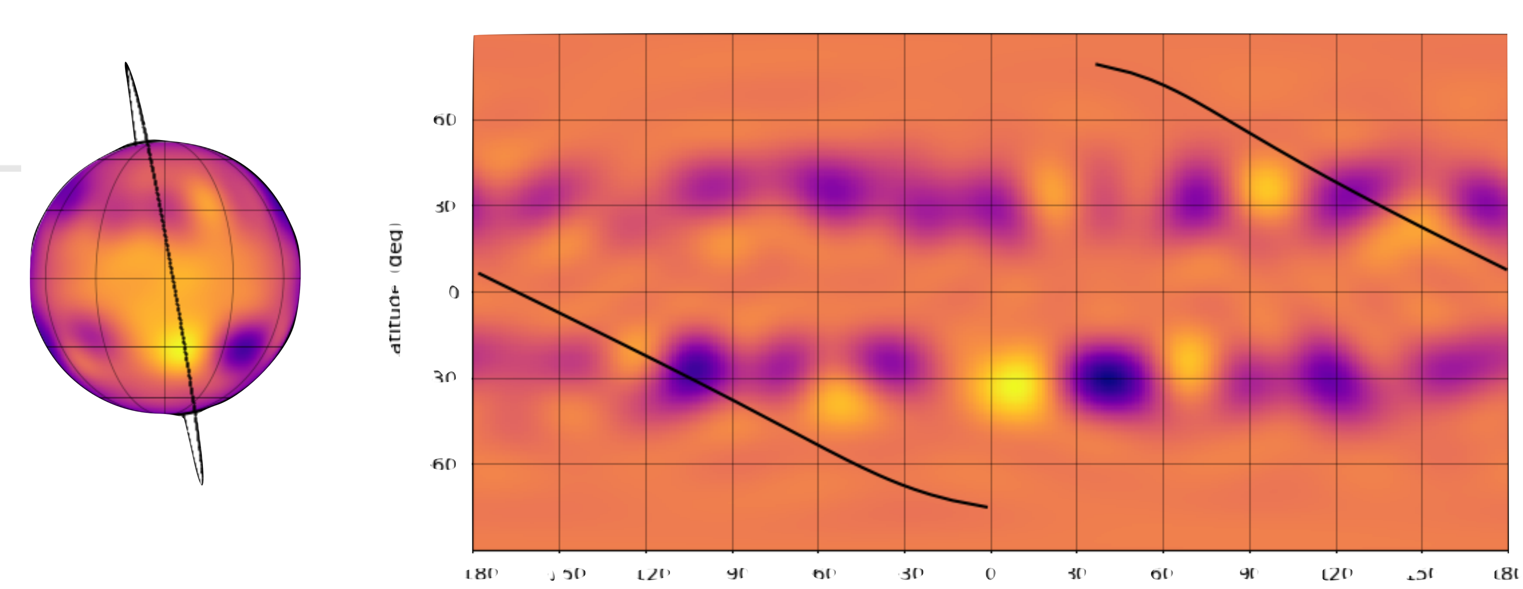
$$p(\Theta, \mathbb{f}_{true} \mid \mathbb{f}_{obs}) \propto p(\Theta) p(\mathbb{f}_{true} \mid \Theta) p(\mathbb{f}_{obs} \mid \mathbb{f}_{true})$$

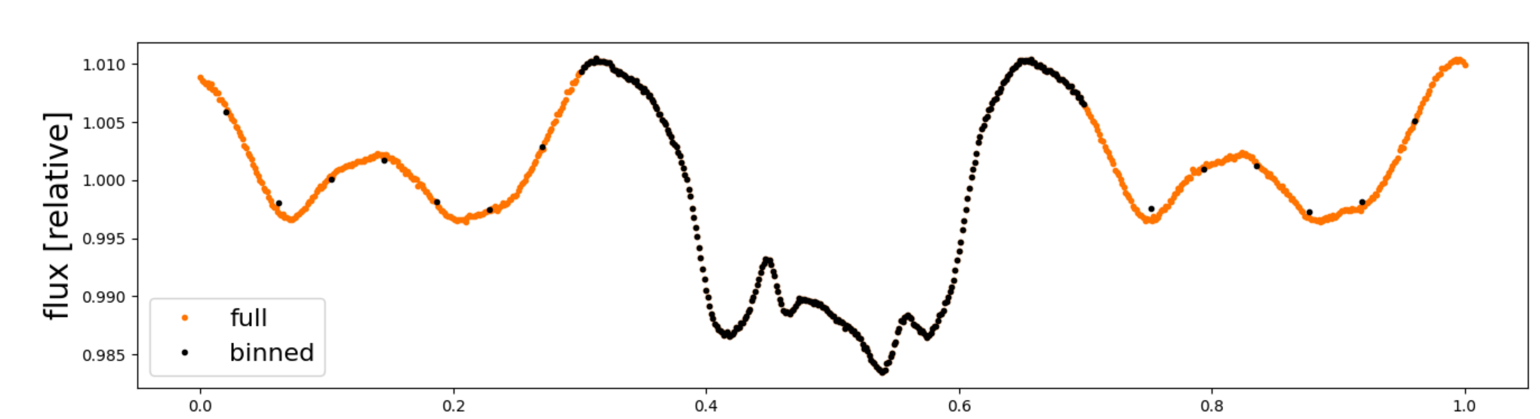
We calculate the log-likelihood function, given by:

$$\ln p(f_{obs} \mid \Theta) = -\frac{1}{2}(f_{obs} - \mu)^T \Sigma^{-1} (f_{obs} - \mu) - \frac{1}{2} \ln |\Sigma| - \frac{n}{2} \ln 2\pi$$

Results. Synthetic light curve:

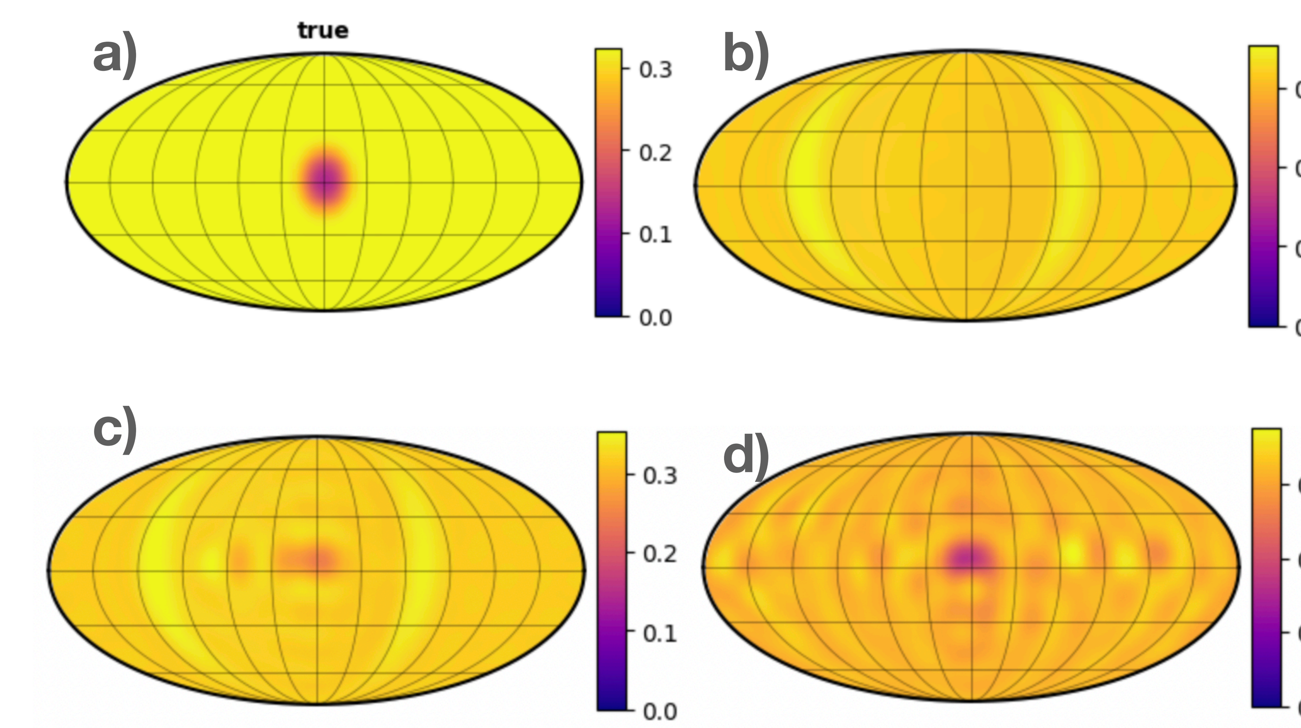
In this experiment, we have a synthetic light curve with one transit. Here, we solve for the spots parameters (the corner plot is shown on the right).



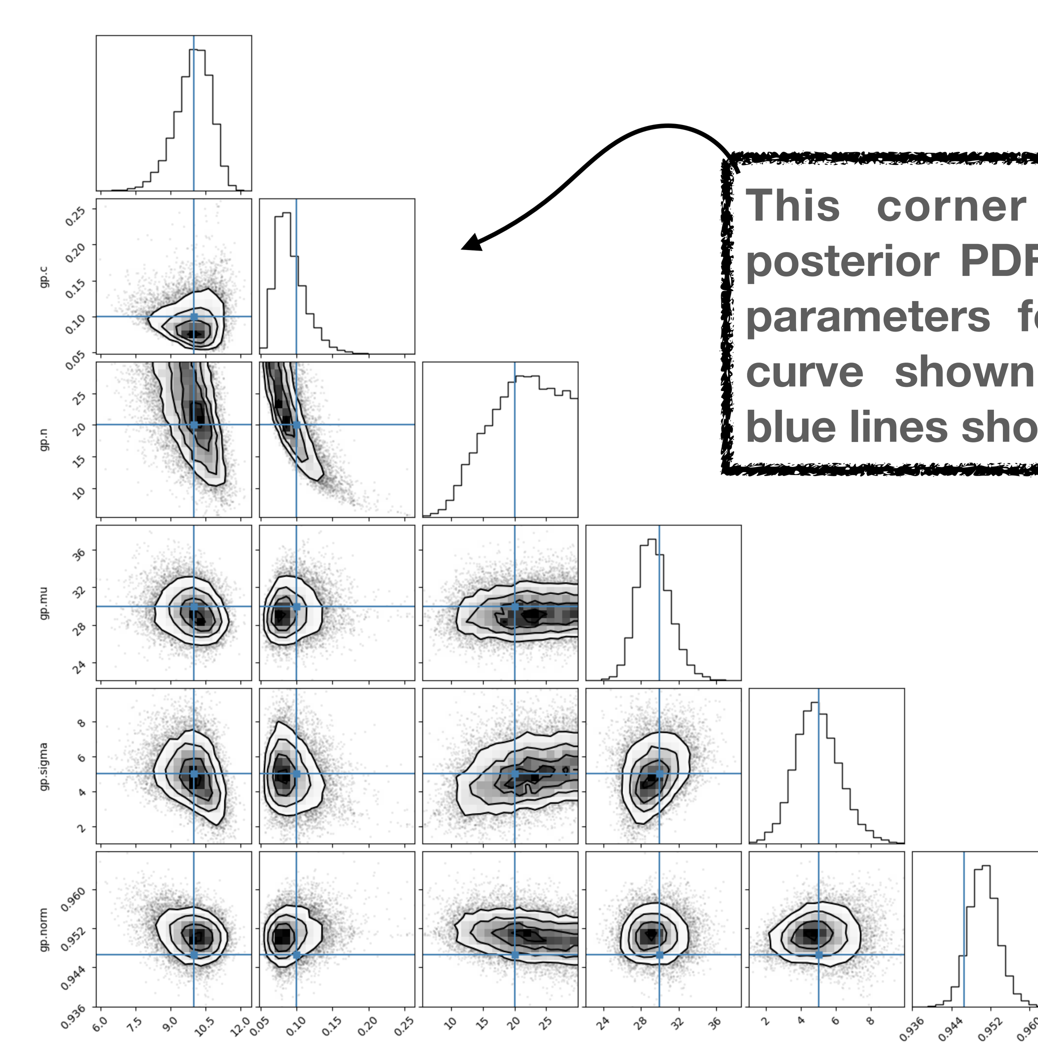


The synthetic light curve with one transit crossing multiple spots. The black points show the light curve with binning the out-of-transit points. The orange points show the full light curve.

Here, we do inference on a map with a star with only one spot to see planetary transit helps to constrain that spot.



Results of L2 regularization inference in pixel space. a) the true map; b) inferred map with no planet; c) inferred map with one transit; d) inferred map with 10 transits. It shows that more transits give us better constraints on the parameters of spots.



This corner plot shows the posterior PDFs for all star spot parameters for the mock light curve shown on the left. The blue lines show the true values.

References:
Luger et al. 2021, AJ, 162, 123
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